A note on the series $\sum L_k(x)/(k+w)$

M.L. Glasser

Department of Theoretical Physics
Universidad de Valladolid
Valladolid, Spain 47011

The Laguerre series

$$S(x,w) = \lim_{n\to\infty} \sum_{k=0}^{n} \frac{L_k(x)}{k+w} \tag{1}$$

appears to have been first applied in representing the Green function for an electron in a uniform magnetic field by R.Kubo et al.[1] in 1965. It is now encountered quite frequently in this connection, especially with respect to electrons in the currently active area of two-dimensional Dirac-Weyl systems such as Graphene and transition metal dichalcogenides[2-6,8]. This may be unfortunate, as for large index $L_k(x) = O(k^{-1/4})$[7] rendering S slowly convergent so truncation is problematic.Thus, the results of numerical summation could be misleading. This use of the series is surprising since the exact expression[8]

$$\sum_{k=0}^{\infty} \frac{L_k^{(-a)}(x)}{k+w} = \Gamma(w)U(w, a+1, x) \tag{2}$$

where U the Tricomi function, goes back to the 19-th century and special cases have been rediscovered repeatedly[10-15].

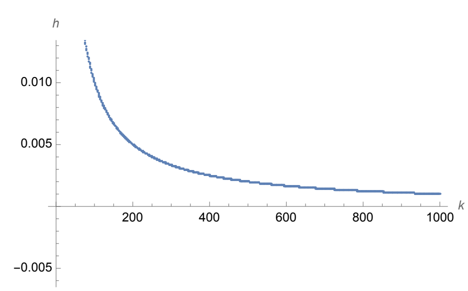

Figure 1: First 1000 terms of $S(-.456, -.777)$)

As an example, we examine $S(-0.456, -0.777)$ .whose value from (11) is 1.872712344.... In Fig.1 are shown the first 1000 terms of $S(.456, .777)$ and by direct summation the values

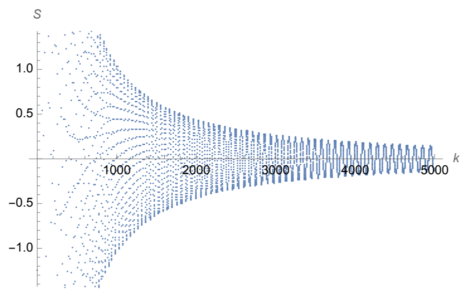

Figure 2: First 1000 terms of $S(20, -.8)$)

1.874280624 for 2000 terms

1.871062815 for 5000 terms

1.872302129. for 10000 terms

1.872508965. for 20000 terms

have been obtained. These values indicate that even summing over 5000 terms may yield the correct result to only two places and even quadrupling the number does not substantially improve the accuracy. Fig.2 shows 5000 terms for $S(20, -.8)$. The recent study[8], involving this truncation, presents results to 5 decimal places.

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