General slow-roll inflation in $f(R)$ gravity under the Palatini approach

Sabit Bekov,[1,2] Kairat Myrzakulov,[1,2] Ratbay Myrzakulov,[1,2] and Diego Sáez-Chillón Gómez[1,3,*]

[1] *Eurasian International Center for Theoretical Physics, Nur-Sultan, 010009, Kazakhstan*
[2] *Eurasian National University, Nur-Sultan, 010008, Kazakhstan*
[3] *Department of Theoretical Physics, Atomic and Optics, Campus Miguel Delibes, University of Valladolid UVA, Paseo Belén, 7, 47011 - Valladolid, Spain*

Slow-roll inflation is analyzed in the context of modified gravity within the Palatini formalism. As shown in the literature, inflation in this framework requires the presence of non-traceless matter, otherwise it does not occur just as a consequence of the non-linear gravitational terms of the action. Nevertheless, by including a single scalar field that plays the role of the inflaton, slow-roll inflation can be performed in these theories, where the equations lead to an effective potential that modifies the dynamics. We obtain the general slow-roll parameters and analyze a simple model to illustrate the differences introduced by the gravitational terms under the Palatini approach, and the modifications on the spectral index and the tensor to scalar ratio predicted by the model.

I. INTRODUCTION

General Relativity and other theories constructed from scalar invariants of the Riemann and Ricci tensors assume a metric compatible connection, the so-called Levi-Civita connection, which states that $\nabla_\lambda g_{\mu\nu} = 0$ and the gravitational action is constructed in terms of such connection and its derivatives. Nevertheless, one may consider both objects in principle as independent fields at the level of the action and then, applying the variational principle to obtain the field equations for both, this is the so-called Palatini approach. In particular, when a non-linear function of the Ricci scalar is considered, where the Ricci tensor is constructed in terms of an arbitrary connection, the theory does not lead to General Relativity or the usual metric $f(R)$ equations, but to different ones with diverse phenomenology (for a review see [1]).

The so-called Palatini $f(\mathcal{R})$ gravity, where $\mathcal{R} = g^{\mu\nu}\mathcal{R}_{\mu\nu}(\Gamma)$ with Γ a independent field, has been widely analyzed in the literature. Specifically, Palatini gravity have been studied in the context of dark energy models and late-time constraints [2], where some models show a very promising behavior. Moreover, the modifications on the energy conditions have been also studied in [3], and the possible violations of the local gravity tests have been analyzed with the post-Newtonian approach applied to this type of theories [4], as well as the stellar structure [5] and the junction conditions [6], where significant and interesting differences are found. In a more analytic context, the variational principle [7], the Cauchy problem [8] and the Birkhoff's theorem have been explored [9], also a possible mapping among solutions in GR and these theories is given in [10]. In addition, non-singular black hole solutions seem to be very common in the context of the Palatini formalism [11] as well as regular wormholes solutions [12]. Also the Palatini formalism is usual in the context of Born-Infeld actions [13].

On the other hand, the inflationary paradigm, that solves some of the most important theoretical problems of the Big Bang model, is still being deeply analyzed, with many models that reproduces such initial super-accelerating phase [14]. In addition, inflation also produces the required fluctuations for the perturbations that form the seeds on the variation of the matter distribution that formed the galaxies and clusters of galaxies as well as the anisotropies in the Cosmic Microwave Background (CMB) [15]. Most of the inflationary models are constructed in terms of scalar fields with the appropriate potential, which generally provides a slow-roll behavior of the scalar field that leads to an exponential expansion at the beginning of inflation and then the field rolls down, leading finally to the reheating regime at the end of inflation [16]. These models are constructed in such a way that can satisfy the constraints coming from the analysis of the data collected by Planck over the last years [17]. Nevertheless, inflation have been also well studied within other underlying models, as in the so-called $f(R)$ gravities [18], where the R^2 model or Starobinsky model [19] shows one of the best behaviors when comparing to the observational data.

The present paper is aimed to analyze the slow-roll scenario in the framework of $f(\mathcal{R})$ gravity under the Palatini approach [20]. Inflation has been already studied in the context of Palatini approach, where a special emphasis has

*Electronic address: diego.saez@uva.es

been done in extending the Starobinsky model of metric $f(R)$ gravity to the Palatini formalism, where besides the proper gravitational terms in the action, an inflaton scalar field is considered and its modified properties are analysed in the Einstein frame [21]. In addition, reheating in within this $\mathcal{R}^2$ models has also been studied [22]. Moreover, some particular models as Higgs inflation [23] and Coleman-Weinberg inflation [24]. Here, we aim to obtain the general slow-roll condition with an arbitrary $f(\mathcal{R})$ action in the Palatini formalism with the presence of an scalar field that plays the role of the inflaton that slow rolls, i.e. its kinetic term is very small in comparison to its potential at the beginning of inflation. This analysis is performed completely in the Jordan frame, where we find that the auxiliary scalar field associated to the gravitational part of the action, and which does not propagate, introduces modifications on the potential of the inflaton, leading to an effective potential that modifies the slow-roll parameters and consequently the spectral index and the tensor to scalar ratio.

The paper is organized as follows: in section II, we introduce the Palatini formalism in $f(\mathcal{R})$ gravity theories. Section III reviews the main features of inflation in metric theories and the impossibility of extending such features to the Palatini formalism in vacuum. Then, in section IV we obtain the general slow-roll parameters in these theories with the presence of a scalar field that plays the role of the inflaton. Finally, in V we present the conclusions of the paper.

II. $f(R)$ PALATINI GRAVITY

Let us start by reviewing the main equations and features of $f(R)$ gravity a la Palatini. The general gravitational action is given by:

$$S = \frac{1}{2\kappa^2}\int d^4x\sqrt{-g}f(\mathcal{R}) + S_m \ , \tag{1}$$

where the matter action S_m just depends on the metric and the matter fields, preserving the Equivalence Principle, whereas the Ricci scalar $\mathcal{R}$ is defined as follows:

$$\mathcal{R} = g^{\mu\nu}\mathcal{R}_{\mu\nu}(\Gamma) = g^{\mu\nu}\left(\partial_\lambda\Gamma^\lambda_{\mu\nu} - \partial_\nu\Gamma^\lambda_{\mu\lambda} + \Gamma^\lambda_{\sigma\lambda}\Gamma^\sigma_{\mu\nu} - \Gamma^\lambda_{\sigma\nu}\Gamma^\sigma_{\mu\lambda}\right) \ . \tag{2}$$

Here Γ is the connection and in principle is an independent field from the metric, such that in order to find the field equations, the action (1) should be varied with respect to the metric and to the connection, leading to the following field equations respectively:

$$\begin{aligned} f_\mathcal{R}\mathcal{R}_{\mu\nu} - \frac{1}{2}g_{\mu\nu}f &= \kappa^2 T_{\mu\nu} \ , \\ \nabla_\lambda\left(\sqrt{-g}f_\mathcal{R}g^{\mu\nu}\right) &= 0 \ , \end{aligned} \tag{3}$$

where $T_{\mu\nu} = -\frac{2}{\sqrt{-g}}\frac{\delta S_m}{\delta g^{\mu\nu}}$ is the energy-momentum tensor that only depends on the matter fields and the metric, while $f_\mathcal{R} = \frac{df}{d\mathcal{R}}$. Note also that we have not assumed any symmetry on the indexes of the connection, so this might own an antisymmetric part. However, by the proyective invariance of the scalar curvature, the Ricci scalar itself only depends on the symmetric part of the connection $R(\Gamma)$ and consequently Γ is symmetric under the low indexes. By taking the trace of the first equation in (3), it yields:

$$f_\mathcal{R}\mathcal{R} - 2f = \kappa^2 T \ , \tag{4}$$

Hence, this equation provides an algebraic equation for the scalar curvature $\mathcal{R}$ that can be solved as a function of the trace of the energy-momentum tensor $\mathcal{R} = \mathcal{R}(T)$. In addition, the second equation in (3) is the metricity condition for the conformal metric:

$$h_{\mu\nu} = \Omega^2 g_{\mu\nu} \ , \quad \Omega^2 = f_\mathcal{R} \ , \tag{5}$$

such that the second equation in (3) turns out:

$$\nabla_\lambda\left(\sqrt{-h}h^{\mu\nu}\right) = 0 \ . \tag{6}$$

As pointed above, the solution of this equation leads to a connection that is given by the Christoffel symbols in terms of the metric $h_{\mu\nu}$. Then, by applying the conformal transformation (5), the Ricci tensor $\mathcal{R}_{\mu\nu}$ becomes:

$$\mathcal{R}_{\mu\nu}(h) = R_{\mu\nu}(g) + \frac{4}{\Omega^2}\nabla_\mu\Omega\nabla_\nu\Omega - \frac{2}{\Omega}\nabla_\mu\nabla_\nu\Omega - g_{\mu\nu}\frac{g^{\rho\sigma}}{\Omega^2}\nabla_\rho\Omega\nabla_\sigma\Omega - g_{\mu\nu}\frac{\Box\Omega}{\Omega} \ . \tag{7}$$

Here the covariant derivatives are defined in terms of the Levi-Civita connection of the metric g. Hence, the first field equation in (3) can be written as follows:

$$R_{\mu\nu}(g)-\frac{1}{2}g_{\mu\nu}R(g)=\frac{\kappa^2}{f_{\mathcal{R}}}T_{\mu\nu}-g_{\mu\nu}\frac{\mathcal{R}f_{\mathcal{R}}-f}{2f_{\mathcal{R}}}-\frac{3}{2f_{\mathcal{R}}^2}\left[\nabla_\mu f_{\mathcal{R}}\nabla_\nu f_{\mathcal{R}}-\frac{1}{2}g_{\nu\mu}\nabla_\lambda f_{\mathcal{R}}\nabla^\lambda f_{\mathcal{R}}\right]+\frac{1}{f_{\mathcal{R}}}\left[\nabla_\mu\nabla_\nu f_{\mathcal{R}}-g_{\mu\nu}\Box f_{\mathcal{R}}\right]\ . \tag{8}$$

This equation together with the trace equation (30) form the set of field equations for $f(\mathcal{R})$ in the Palatini formalism that can be applied to study any spacetime in a simple way. Note that the right hand side of (8) depends solely on the energy-momentum tensor and its trace, since $\mathcal{R}=\mathcal{R}(T)$ by the equation (30). Moreover, the field equations (8) are equivalent to a scalar-tensor theory described by the action:

$$S=\frac{1}{2\kappa^2}\int d^4x\sqrt{-g}\left[\phi R(g)+\frac{3}{2\phi}\partial_\mu\phi\partial^\mu\phi-V(\phi)\right]+S_m\ , \tag{9}$$

where the correspondence is given by:

$$\phi=f_{\mathcal{R}}\ ,\quad V(\phi)=\mathcal{R}\phi-f(\mathcal{R})\ . \tag{10}$$

The Lagrangian (9) is the action for Brans-Dicke theory with a potential and with the usual parameter given by $w=-3/2$.

III. INFLATION IN $f(R)$ GRAVITIES

Let us first review the main achievements in metric $f(R)$ gravity when describes the inflationary paradigm. In this sense, the so-called Starobinsky inflation [19] represents the main success of implementing inflation within the modified gravity framework. The model is based on the presence of a quadratic term of the Ricci scalar in the action, which is usually given by:

$$S=\frac{1}{2\kappa^2}\int \mathrm{d}^4x\sqrt{-g}\left[R+\frac{R^2}{6\,m^2}\right]\ . \tag{11}$$

Recall that the Ricci scalar in (11) depends on the Levi-Civita connection and consequently the only fundamental field here is the metric tensor. As under the Palatini approach, this action can be expressed in terms of a scalar field:

$$S=\frac{1}{2\kappa^2}\int \mathrm{d}^4x\sqrt{-g}\left[\phi R-V(\phi)\right]\ . \tag{12}$$

Here as above, $\phi=f_{\mathcal{R}}$ and $V(\phi)=R\phi-f(R)$. However, the action (12) has no kinetic term for the the scalar field, in comparison to (9). Nevertheless, this does not avoid the scalar field in (12) to propagate, as shown by the trace of the field equations for this action, which reads:

$$3\Box\phi-R\phi+2V(\phi)=\kappa^2T\ . \tag{13}$$

In general, the inflationary model described in metric $f(R)$ gravities is analysed in the Einstein frame instead of studying the action (12) directly. By performing the conformal transformation:

$$g_{\mu\nu}\quad\longrightarrow\quad\Omega^2 g_{\mu\nu}\ , \tag{14}$$

the action (12) becomes:

$$\tilde{S}=\int \mathrm{d}^4x\sqrt{-g_E}\left[\frac{R_E}{2\kappa^2}-\frac{1}{2}\partial_\mu\phi_E\partial^\mu\phi_E-V_E(\phi_E)\right]\ , \tag{15}$$

where the subindex $_E$ refers to the quantities transformed to the Einstein frame. The potential $V_E(\phi_E)$ for the Starobinsky action (11) turns out:

$$V_E(\phi_E)=\frac{3}{4\kappa^2}m^2\left(1-e^{-\sqrt{2/3}\kappa\phi_E}\right)^2. \tag{16}$$

We assume a flat FLRW metric in the Einstein frame:

$$ds^2=-dt^2+a(t)^2\sum_i dx_i^2\ . \tag{17}$$

The corresponding FLRW equations lead to:

$$H^2 = \frac{\kappa^2}{3}\left(\frac{1}{2}\dot{\phi}_E^2 + V_E(\phi_E)\right) \ , \quad \dot{H} = -\frac{\kappa^2}{2}\dot{\phi}_E^2 \ , \tag{18}$$

while the equation for the scalar field ϕ is obtained by varying the action (15) with respect to the scalar field:

$$\ddot{\phi}_E + 3H\dot{\phi}_E + V'(\phi_E) = 0 \ . \tag{19}$$

Hence, the FLRW equations (20) together with the scalar field equation (19) may correspond to inflationary models with a single scalar field minimally coupled. The slow-roll scenario occurs in the regime $\frac{1}{2}\dot{\phi}_E^2 << V_E(\phi_E)$ and $\ddot{\phi}_E << H\dot{\phi}_E$, which approximates the first FLRW equation and the scalar field equation to:

$$H^2 \approx \frac{\kappa^2}{3}V_E(\phi_E) \ , \quad 3H\dot{\phi}_E + V'(\phi_E) \approx 0 \ . \tag{20}$$

This is the slow-roll approximation, which is equivalent to the following conditions on the so-called slow-roll parameters:

$$\epsilon = -\frac{\dot{H}}{H^2} \approx \frac{1}{2\kappa^2}\left(\frac{V_E'}{V}\right)^2 << 1 \ , \quad |\eta| \approx \frac{1}{\kappa^2}\left(\frac{V_E''}{V_E}\right) << 1 \ . \tag{21}$$

Hence, for the appropriate scalar potential, the slow-roll inflationary scenario can be implemented. Actually this is the case for the potential (16), whose slow-roll parameters become:

$$\epsilon \ \simeq \ \frac{4}{3}\left(e^{\sqrt{\frac{2}{3}}\kappa\phi_E} - 1\right)^{-2} \simeq \frac{4}{3}e^{-2\sqrt{\frac{2}{3}}\kappa\phi_E} \ , \tag{22}$$

$$\eta \ \simeq \ \frac{4}{3}\frac{2 - e^{\sqrt{\frac{2}{3}}\kappa\phi_E}}{\left(-1 + e^{\sqrt{\frac{2}{3}}\kappa\phi_E}\right)^2} \simeq -\frac{4}{3}e^{-\sqrt{\frac{2}{3}}\kappa\phi_E}. \tag{23}$$

Here we have assumed that the scalar field is large enough at the beginning of inflation $\kappa\phi_E >> 1$, which makes $\epsilon << 1$ and $\eta << 1$, and the Hubble parameter is approximately a constant, which leads to an exponential expansion. Inflation ends when the scalar field rolls down and its kinetic term becomes larger, what makes $\epsilon \approx 1$. The important point of inflation is to last enough in order to solve the initial conditions problems, such that it should last a sufficient number of e-foldings:

$$N = \int_{t_{start}}^{t_{end}} H dt \approx 55 - 65 \ . \tag{24}$$

By using the FLRW equation, this can be computed in terms of the scalar field:

$$N = -\kappa^2 \int_{\phi_{Ei}}^{\phi_{Eend}} \frac{V_E(\phi_E}{V_E'(\phi_E)} \mathrm{d}\phi_E \approx \frac{3}{4}e^{\sqrt{2/3}\kappa \ \phi_{Ei}} \ . \tag{25}$$

Note that the number of e-foldings is related to the slow-roll parameters as follows:

$$\epsilon \simeq \frac{3}{4}\frac{1}{N^2} \ , \quad \eta \simeq -\frac{1}{N} \ . \tag{26}$$

While the spectral index for curvature perturbations and the tensor to scalar ratio are given by:

$$n_s - 1 = -6\epsilon + 2\eta \ , \quad r = 16\epsilon \tag{27}$$

The last constraints on both parameters from Planck [17] are:

$$n_s = 0.9659 \pm 0.0041 \ , \quad r < 0.11 \ . \tag{28}$$

Hence, for the Starobinsky inflation by assuming $N = 65$, we obtain:

$$n_s = 0.968 \ , \quad r = 0.00284 \, , \tag{29}$$

which fit very well the above constraints and this is one of the great success of Starobinsky inflation and metric $f(R)$ inflation in general.

Nevertheless, this is not the case for Palatini $f(\mathcal{R})$ gravity. From the trace equation (30), we can obtain the equation for the scalar field ϕ described by the action (9):

$$2V(\phi) - \phi \frac{dV}{d\phi} = \kappa^2 T \ . \tag{30}$$

Here, one directly notes that in comparison to the metric case (13), the scalar field does not propagate, and in absence of matter, the scalar field becomes a constant, such that it only provides and effective cosmological constant, which produces an accelerating expansion with no end and obviously with no perturbations. Hence, the only way to produce inflation in Palatini $f(\mathcal{R})$ gravity is in the presence of non-traceless matter, as the presence of a scalar field that plays the role of the inflaton, inducing modifications on the slow-roll scenario, as is shown in the next section.

IV. SLOW-ROLL INFLATION IN PALATINI $f(\mathcal{R})$ GRAVITY

As pointed above, since the scalar field associated to the $f(\mathcal{R})$ action in the Palatini approach, does not propagate and its source is the presence of non traceless matter, the only way to reproduce inflation is to include an inflaton field. Nevertheless, as will be shown below, the slow-roll conditions are modified in comparison to General Relativity when a Palatini $f(\mathcal{R})$ Lagrangian is assumed. Then, here we will analyze the Palatini $f(\mathcal{R})$ Lagrangian in its version expressed in terms of an auxiliary scalar field as given in (9), with the presence of an additional scalar field χ that plays the role of the inflaton:

$$S = \frac{1}{2\kappa^2}\int d^4x\sqrt{-g}\left[\phi R(g) + \frac{3}{2\phi}\partial_\mu\phi\partial^\mu\phi - V(\phi)\right] + \int d^4x\sqrt{-g}\left[-\frac{1}{2}\partial_\mu\chi\partial^\mu\chi - U(\chi)\right] \ . \tag{31}$$

The corresponding field equations are given by (8), where recall that $\phi = f_{\mathcal{R}}$ and $V(\phi) = \mathcal{R}\phi - f(\mathcal{R})$ and the energy-momentum tensor χ:

$$T^{\chi}_{\mu\nu} = \partial_\mu\chi\partial_\nu\chi - g_{\mu\nu}\left(\frac{1}{2}\partial_\alpha\chi\partial^\alpha\chi + U(\chi)\right) \ . \tag{32}$$

By assuming a flat FLRW metric as given in (17), the FLRW equations yield:

$$\begin{aligned} 3H^2 &= \frac{\kappa^2}{\phi}\left(\frac{1}{2}\dot{\chi}^2 + U(\chi)\right) - 3H\frac{\dot{\phi}}{\phi} - \frac{3}{4}\left(\frac{\dot{\phi}}{\phi}\right)^2 + \frac{1}{2}V(\phi) \ , \\ -2\dot{H} &= \frac{\kappa^2}{\phi}\dot{\chi}^2 - H\frac{\dot{\phi}}{\phi} - \frac{3}{2}\left(\frac{\dot{\phi}}{\phi}\right)^2 + \frac{\ddot{\phi}}{\phi} \ . \end{aligned} \tag{33}$$

While the equations for the scalar fields are given by:

$$\begin{aligned} -2V(\phi) + \phi V'(\phi) &= \kappa^2\left(\dot{\chi}^2 - 4U(\chi)\right) \ , \\ \ddot{\chi} + 3H\dot{\chi} + U'(\chi) &= 0 \ . \end{aligned} \tag{34}$$

We can now consider the scalar field χ to be in the slow roll regime during the inflationary phase, as in usual single field inflation in GR. Then, the following slow-roll conditions are assumed on χ:

$$U(\chi) >> \frac{1}{2}\dot{\chi}^2 \ , \quad H\dot{\chi} >> \ddot{\chi} \ . \tag{35}$$

Hence, the scalar field equations (34) reduce to:

$$\begin{aligned} -2V(\phi) + \phi V'(\phi) &\approx -4\kappa^2 U(\chi) \ , \\ 3H\dot{\chi} + U'(\chi) &\approx 0 \ . \end{aligned} \tag{36}$$

From here we end to the conclusion that the problem is reduced to a single scalar field inflationary model, since from the first equation in (36), one gets $\phi \approx \phi(\chi)$, as far as the equation has real solutions in terms of ϕ. This is not surprising, as equation (34) for ϕ does not contain first or second derivatives of the scalar field, an aspect already discussed in the section above, inherent to the Palatini approach, that avoids the auxiliary scalar field ϕ to propagate. Moreover, as a consequence of $\phi \approx \phi(\chi)$ during inflation, we can directly assume that $\dot{\phi} << \phi$. Hence, the FLRW equations (33) can be expressed just in terms of the inflaton field χ and are approximated as follows:

$$\begin{aligned} 3H^2 &\approx \frac{1}{\phi}\left[\kappa^2 U(\chi) + \frac{d\phi}{d\chi}U'(\chi) + \frac{1}{2}V(\phi(\chi))\right] = \kappa^2 U_{eff}(\chi) \ , \\ -2\dot{H} &\approx \frac{1}{\phi}\left(\kappa^2\dot{\chi}^2 + \frac{1}{3}\frac{d\phi}{d\chi}U'(\chi)\right) \approx \frac{1}{3\phi}\left(\frac{U'(\chi)}{U_{eff}(\chi)} + \frac{d\phi}{d\chi}U'(\chi)\right) \ . \end{aligned} \tag{37}$$

Recall that $\phi = \phi(\chi)$, such that these equations only depend on the dynamics of the inflaton field χ. In addition, here we have used the slow-roll conditions on χ (35) and the scalar field equations (36), while we have defined the effective potential as:

$$U_{eff} = \frac{1}{\kappa^2\phi}\left[\kappa^2 U + \frac{d\phi}{d\chi}U'(\chi) + \frac{1}{2}V(\phi(\chi))\right] \ . \tag{38}$$

Hence, the slow-roll parameters (21) will read now differently:

$$\begin{aligned} \epsilon &\approx \frac{1}{2\kappa^2\phi}\left[\left(\frac{U'}{U_{eff}}\right)^2 + \frac{d\phi}{d\chi}\frac{U'}{U_{eff}}\right] \ , \\ |\eta| &\approx \frac{1}{\kappa^2}\frac{U''}{U_{eff}} \ . \end{aligned} \tag{39}$$

Note that for $f(\mathcal{R}) = \mathcal{R} = R(g)$, i.e. when General Relativity is recovered, the scalar field $\phi = 1$ and the usual slow-roll parameters (21) are recovered in terms of the inflaton potential U. Hence, slow-roll inflation will occur when $\epsilon << 1$ and $\eta << 1$, while inflation ends for $\dot{\chi}^2 \approx 2U(\chi)$, when the approximation (36) is no longer valid.

Let us consider a model to illustrate the differences introduced by the modified gravitational action $f(\mathcal{R})$. To do so, we choose the following -commonly used- potential for the inflaton:

$$U(\chi) = \frac{1}{2}m_\chi^2\dot{\chi}^2 \ . \tag{40}$$

This potential has been widely analysed in the literature and describes the so-called chaotic inflation, which under the slow-roll conditions (35) in General Relativity gives:

$$H^2 \approx \frac{\kappa^2}{6}m_\chi^2\chi^2 \ . \tag{41}$$

While the number of e-foldings (25) can be expressed in terms of the scalar field as follows:

$$N = -\kappa^2\int_{\chi_i}^{\chi_{end}}\frac{U(\chi}{U'(\chi)}\mathrm{d}\chi \approx \frac{\kappa^2\chi_i^2}{4} \ . \tag{42}$$

Hence, the slow-roll parameters (21) are:

$$\epsilon = \eta = \frac{1}{N} \ , \tag{43}$$

which for a number of e-foldings $N = 65$ provides the following values for the spectral index and the tensor to scalar ratio:

$$n_s = 0.97 \ , \quad r = 0.12 \ . \tag{44}$$

By comparing with the last data from Planck (28), it is clear that the r does not satisfy the constraint.

Let us now analyze the same potential for the inflaton (40) in the context of Palatini gravity when assuming the following gravitational action:

$$f(\mathcal{R}) = \alpha\mathcal{R}^n \ , \tag{45}$$

where α is a constant with the appropriate dimensions and n too. The corresponding auxiliary scalar field ϕ and its potential are then provided by (10) and lead to:

$$\phi = n\alpha\mathcal{R}^{n-1} \ , \quad V(\phi) = k\ \phi^{n/n-1} \ , \tag{46}$$

here $k = \frac{1}{n^{1/n-1}} - \alpha$. By assuming the slow-roll conditions for the scalar field χ given in (35), the relation $\phi = \phi(\chi)$ is obtained from the first equation in (36):

$$\phi(\chi) \approx \left(\frac{k(n-2)}{2m_\chi^2(n-1)}\frac{1}{\kappa^2\chi^2}\right)^{\frac{1-n}{n}} \ . \tag{47}$$

From here we can deduce that $n \neq 1, 2$, since for $n = 1$ we recover General Relativity and $n = 2$ is a degenerate case as shown by the trace equation (30). The Hubble parameter in (37) leads to:

$$3H^2 = \frac{\kappa^2 m_\chi^2}{2k\ n(n-2)} \frac{2^{1/n}k\ n(3n-4) + 8m_\chi^2(n-1)^2\left(\frac{k(n-2)}{(n-1)m_\chi^2\kappa^2\chi^2}\right)^{1/n}}{\left(\frac{k(n-2)}{(n-1)m_\chi^2\kappa^2\chi^2}\right)^{\frac{1-n}{n}}}\chi^2 \ , \tag{48}$$

which for the case $n > 2$ and assuming a large field at the beginning of inflation $\kappa\chi >> 1$, it yields:

$$3H^2 \approx \frac{2^{\frac{1-n}{n}}(3n-4)\kappa^2 m_\chi^2}{(n-2)}\left(\frac{k(n-2)}{(n-1)m_\chi^2\kappa^2\chi^2}\right)^{\frac{|1-n|}{n}}\chi^2 \ . \tag{49}$$

In order to simplify the expressions, we can assume for illustrative purposes, the case $n = 3$. Hence, the slow-roll parameters (39) turn out:

$$\epsilon = \frac{5\cdot 2^{4/3}m_\chi^2}{75k}\left(\frac{k}{m_\chi^2\kappa^2\chi^2}\right)^{1/3} \ , \quad \eta = \frac{2^{7/3}m_\chi^2}{5k}\left(\frac{k}{m_\chi^2\kappa^2\chi^2}\right)^{1/3} \ , \tag{50}$$

while the number of e-foldings is given by:

$$N = \int_{t_i}^{t_{end}} Hdt = -\kappa^2\int_{\chi_i}^{\chi_{end}}\frac{U_{eff}}{U'}d\chi = \frac{15}{8\cdot 2^{1/3}}\left(\frac{k}{m_\chi^2}\right)^{2/3}(\kappa^2\chi_i^2)^{1/3} \ . \tag{51}$$

Then, the slow-roll parameters (50) can be expressed in terms of the number of e-foldings as follows:

$$\epsilon = \frac{13}{10}\frac{1}{N} \ , \quad \eta = \frac{3}{2}\frac{1}{N} \ , \tag{52}$$

While the spectral index and the tensor to scalar ratio lead to:

$$n_s = 0.92 \ , \quad r = 0.34 \ , \tag{53}$$

which neither satisfy the constraints from Planck (28). Actually, any integer for $n > 2$ does not provide good fits to Planck data. Nevertheless, for half integers, the predictions are different. We may consider $n = 4/5$, in whose case the Hubble parameter yields:

$$\epsilon = \frac{3^{5/4}m_\chi^2}{4k}\left(\frac{k}{m_\chi^2\kappa^2\chi^2}\right)^{5/4} \ , \quad \eta = \frac{3^{5/4}m_\chi^2}{2k}\left(\frac{k}{m_\chi^2\kappa^2\chi^2}\right)^{5/4} \ , \tag{54}$$

whereas the number of e-foldings is given by:

$$N = \frac{4}{5\cdot 3^{5/4}}\left(\frac{m_\chi^2}{k}\right)^{1/4}(\kappa^2\chi_i^2)^{5/4} \ . \tag{55}$$

Hence, the slow-roll parameters in terms of N gives:

$$\epsilon = \frac{1}{5}\frac{1}{N} \ , \quad \eta = \frac{2}{5}\frac{1}{N} \ , \tag{56}$$

which for $N = 65$, we obtain the following spectral index and tensor to scalar ratio:

$$n_s = 0.98 \ , \quad r = 0.05 \ , \tag{57}$$

which fit much better the Planck constraints, particularly the tensor to scalar ratio. Hence, as illustrated by this simple model of chaotic inflation and a power gravitational action, the corrections introduced in the Palatini formalism play an important role in the viability of a particular inflaton model.

V. CONCLUSIONS

In the present paper we have analysed the inflationary paradigm in the framework of $f(\mathcal{R})$ Palatini gravity. As shown in other previous papers, and as a consequence of the non-propagating scalar field in the Palatini formalism, in vacuum or with the presence of traceless matter, the action $f(\mathcal{R})$ just adds an effective cosmological constant that is not enough to construct a viable model for inflation, since inflation should end after a particular number of e-foldings. Hence, the only way is to consider the construction of inflationary models in this framework is to consider the presence of matter, in general a single scalar field with an appropriate potential, similarly as in General Relativity.

Nevertheless, the presence of non-linear functions of the Ricci scalar $\mathcal{R}$ in the gravitational action induces modifications on the dynamics of the Hubble parameter and consequently on the inflaton, leading to an effective potential. Here, by working in the Jordan frame, we have obtained such effective potential and have calculated the modified slow-roll parameters when one assumes a slow-roll motion for the inflaton. As a consequence of these modifications, the corresponding spectral index and tensor to scalar ratio are modified too, such that the same potential for the inflaton does not provide the same predictions in GR and in $f(\mathcal{R})$ Palatini gravity. We have illustrated this feature by a simple case, the so-called chaotic inflation which is described by a quadratic potential for the inflaton. As shown, this model that does not fit well the Planck constraints, is slightly modified by a non-linear term of the Ricci scalar in the action, leading to much better predictions for the spectral index and the tensor to scalar ratio.

Hence, slow-roll inflation in modified gravities within the Palatini approach reflects important differences that may help to satisfy the observational constraints for some models that in the context of GR were ruled out.

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