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Some numerical estimations of energy levels on a model for a graphene ribbon in a magnetic field

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ABSTRACT

We use the averaging method in order to obtain the values of the energy levels of a graphene ribbon under a magnetic field with some given conditions. We contrast the efficiency of the method with other methods for two different choices of the magnetic field.

1. Introduction

The need of numerical methods to obtain the energy spectrum of a given Hamiltonian H is out of question since in most of cases it cannot be obtained explicitly by pure analytical methods. Usually the evaluation of the eigenvalues of H can be done through the numerical resolution of a problem of Sturm–Liouville type. In the present article, we use a version of the averaging method [1] for non-periodic systems [2] in order to perform a numerical determination of the eigenvalues of a system with discrete spectrum. We show evidence that this method is rather effective, at least for low energy levels.

In fact, we use this method for the numerical determination of the spectrum in a system which is not governed by the Schrödinger equation by instead by a slightly different one, the Dirac–Weyl equation applied to the graphene, where as in a typical Sturm–Liouville problem, boundary conditions will also be given [3].

The graphene is a two dimensional layer of graphite, which has received an enormous attention as is expected to be an appropriate material to develop electronic devices [4–6]. From a physical point of view, the interest is focused in the study of the behavior of electrons in graphene strips or ribbons. This study is performed by assuming that the behavior of each singular electron is governed by a massless Dirac equation.

Motivated by the analysis of physical properties of the electrons confined in graphene ribbons, a model of graphene electrons on perpendicular magnetic field has been considered by several authors [7–9]. In these models for the graphene, an electron confined in a graphene ribbon moves with an effective Fermi speed of $v_F = c/300$, where c is the speed of light in the vacuum. Its effective Hamiltonian has the form [5]:

$$H = v_F \boldsymbol{\sigma} \cdot \left(\mathbf{p} + \frac{\mathbf{e}}{\mathbf{c}}\mathbf{A}\right), \tag{1}$$

where $\boldsymbol{\sigma} = (\sigma_x, \sigma_y)$, σ_x and σ_y being the Pauli matrices, e the electron charge, $\mathbf{A} = (A_x(x,y),\ A_y(x,y), 0)$ the potential vector due to an external magnetic field and $\mathbf{p} = -i\hbar(\partial_x, \partial_y)$, the two dimensional momentum operator. Note that, since the magnetic field is $\mathbf{B} = \nabla \times \mathbf{A}$, then it has only a nonvanishing z-component, which we are assuming to be perpendicular to the ribbon. Under the mentioned conditions, the behavior of one electron is governed by an equation of Dirac–Weyl type, which is:

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Regular Article

Superintegrable Lissajous systems on the sphere

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Abstract. The kind of systems on the sphere, whose trajectories are similar to the Lissajous curves, is studied by means of one example. The symmetries are constructed following a unified and straightforward procedure for both quantum and classical versions of the model. In the quantum case it is stressed how the symmetries give the degeneracy of each energy level. In the classical case it is shown how the constants of motion supply the orbits, the motion and the frequencies in a natural way. The associated sets of polynomial constants of motion and symmetries as well as their algebraic relations are also obtained and compared for the classical and quantum cases.

1 Introduction

The well-known Lissajous curves in two dimensions (2D) correspond to the motion of a system that can be described by two Cartesian coordinates, where each coordinate is harmonic in time and the ratio of both frequencies is a rational number. Such curves are closed in a rectangle and depend on the phase difference at the initial time. In a similar way, we want to extend this point of view to systems on the sphere where the motion is obtained in terms of the spherical coordinates $\varphi(t)$ and $\theta(t)$. In this case, the motion in each coordinate will be periodic but not harmonic, and their trajectories closed curves inscribed in a "spherical rectangle", $\varphi_1 \leq \varphi \leq \varphi_2$, $\theta_1 \leq \theta \leq \theta_2$, when the proportion of the two periods is a rational number.

The type of systems leading to Lissajous-type trajectories include the TTW [1,2] and other similar systems [3–11]. They are superintegrable and the search of their symmetries has been the subject of a considerable number of recent contributions. In this work, by means of one example, we want to stress the features of such systems defined on the sphere leading to Lissajous-like curves. In particular, it will be shown how the trajectories are determined and how the period of the motion in each coordinate can be computed by means of pure algebraic methods. We will adopt a simple procedure presented in [12] in order to deal with the symmetries and constants of motion.

Let us emphasize the main novelties of our approach: i) The method to find the symmetries for both quantum and classical versions of these systems is the same. In previous references, quite different procedures were applied in order to obtain classical or quantum symmetries; as a consequence, the origin of the close relationship of classical and quantum algebraic relations was hidden from the very beginning. ii) Our approach is based on the factorization properties of one-dimensional systems [13–15]. This technique, well known in quantum mechanics, is extended to classical systems. For instance, we can define in a straightforward way "ladder functions" and "shift functions", whose counterpart operators are familiar in quantum mechanics. iii) As we will see later, the classical version of our 2D system is maximally superintegrable, so that we can find three independent constants of motion (with two of them in involution). These constants will characterize the orbits. However, one additional constant of motion, depending explicitly on time, will be obtained from some "ladder functions". This additional constant of motion will determine the motion of the system along each orbit. In summary, we have a kind of "complete superintegrability" where the number of independent integrals of motion is $(2n-1)+1 = 2n$ characterizing completely the motion (not only the trajectory).

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Carbon nanotubes in an inhomogeneous transverse magnetic field: exactly solvable model

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Abstract
A class of exactly solvable models describing carbon nanotubes in the presence of an external inhomogeneous magnetic field is considered. The framework of the continuum approximation is employed, where the motion of the charge carriers is governed by the Dirac–Weyl equation. The explicit solution of a particular example is provided. It is shown that these models possess nontrivial integrals of motion that establish $N = 2$ nonlinear supersymmetry in case of metallic and maximally semiconducting nanotubes. Remarkable stability of energy levels with respect to small fluctuations of longitudinal momentum is demonstrated.

Keywords: carbon nanotubes, Dirac equation, magnetic field, finite-gap systems
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(Some figures may appear in colour only in the online journal)

1. Introduction

Despite their structural simplicity, carbon nanotubes possess exciting physical properties [1, 2]. Besides their ultimate strength [3] and elasticity [4], they particularly excel in the variability of their electronic characteristics that make them an attractive material for the applications in electronic devices [5, 6]. Single-wall carbon nanotubes are small cylinders rolled up from a graphene strip with the shell being just one atom-thick. The gap between their valence and conduction band (positive and negative energies) depends on the radius of the nanotube and

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Minimal $D = 4$ supergravity from the superMaxwell algebra

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Abstract

We show that the first-order $D = 4$, $N = 1$ pure supergravity lagrangian four-form can be obtained geometrically as a quadratic expression in the curvatures of the Maxwell superalgebra. This is achieved by noticing that the relative coefficient between the two terms of the lagrangian that makes the action locally supersymmetric also determines trivial field equations for the gauge fields associated with the extra generators of the Maxwell superalgebra. Along the way, a convenient geometric procedure to check the local supersymmetry of a class of lagrangians is developed.

1. Introduction

Since the advent of supersymmetry, there has been an interest in superalgebras going beyond the standard superPoincaré one. For instance, enlarged $D = 11$ supersymmetry algebras were considered by D'Auria and Fré in [1] and further in [2] in a search for the group structure underlying $D = 11$ supergravity [3], which is hidden due to the presence of the three-form that needs being trivialized as a product of one-forms to be associated with Maurer–Cartan (MC) forms. The resulting superalgebras go beyond the $D = 11$ superPoincaré algebra and contain additional fermionic generators and tensorial charges. Larger supersymmetry algebras (and correspondingly enlarged superspaces), also appear associated with super-p-branes, where

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On the entropy for unstable fermionic and bosonic states

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HIGHLIGHTS

- We address the problem of the entropy for complex-energy states.
- The formalism does not require the use of a complex temperature.
- The entropy is calculated starting from a proper definition of the density operator.
- We use complex coherent states and path integrals to compute the density operator.
- The entropy, subject to an interpretation, is bigger for unstable quantum states than for real energy states.

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ABSTRACT

We focus on the calculation of the entropy for decaying states in non-relativistic quantum mechanics. The starting point is the Friedrichs model in second quantization language. In this model, the Hamiltonian admits a spectral representation which includes resonances and Gamow states explicitly. In order to avoid the limitations posed by the definition of canonical probabilities in the presence of a complex spectrum, and/or the use of complex temperatures, we construct the partition function performing a path integration over coherent states. It is shown that the path integration yields results which are correct, at leading order, within the framework of the thermal perturbation theory. Finally, we obtain an expression for the canonical entropy of a quantum decaying system composed of fermion- and boson-states.

1. Introduction

The work of Gamow [1] opened the avenue leading to the quantum mechanical treatment of decaying states, both the mathematics and the physical aspects of resonances have called the attention of a generation of physicists and mathematicians. A concise introduction to the problem may be found in Refs. [2–5]. More recently [6], the subject has been presented in a comprehensive way, to accommodate for applications of the concept to nuclear structure and nuclear reaction problems. So far, the quantum mechanical principles underlying the physics and mathematics of resonances have been consistently formulated in the context of analytic extensions [3] of the standard hermitian eigenvalue problem. A more compact formulation, which is based on Gamow densities, can be found in Ref. [7]. Considerably less effort was devoted to the understanding of the aspects related to the formulation of the problem in the context of the quantum statistical mechanics. A decaying system should also obey thermodynamical laws, in a broad sense not limited by the notion of probabilities, exactly as it happens in the just referred quantum mechanical frame for resonances [3,6]. From the physical point of view

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